P.G. Sem-III,

Functional Analysis

Q - Prove that in a normed linear space $| \|x\| - \|y\| | \leq \|x-y\|$ for all $x, y \in E$

Soln - $\because$ We have

$$x = x - y + y$$

$\therefore$ $\|x\| = \|(x-y)+y\| \leq \|x-y\| + \|y\|$

or, $\|x\| - \|y\| \leq \|x-y\|$ — (1)

Now, By interchanging x and y, we have

$$\|y\| - \|x\| \leq \|y-x\| = \|-(x-y)\|$$

$$= |-1| \, \|(x-y)\|$$

$$= \|x-y\|$$

Ie $\|y\| - \|x\| \leq \|x-y\|$ — (2)

Hence from (1) & (2) we have

$$| \|x\| - \|y\| | \leq \|x-y\|$$

Proved